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MOTION SIMULATION OF GROUND OBSERVED EARTH SATELLITES

J. F. A. Ormsby

June 1969

Prepared for

ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Project 4960

Prepared by

THE MITRE CORPORATION
Bedford, Massachusetts

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FOREWORD

The work reported in this document was performed by The MITRE Corporation, Bedford, Massachusetts, for the Electronic Systems Division of the Air Force Systems Command under Contract AF 19(628)-2390. This information was originally published in Working Paper W-7289, The MITRE Corporation, September, 1964.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

ANTHONY P. TRUNFIO, Technical Advisor
Development Engineering Division
Directorate of Planning and Technology

ABSTRACT

The applicaton of simulation techniques provides a convenient and useful means for studying and developing signature analysis schemes to describe orbiting body configuration, orientation and motion.

This paper discusses a simulation of the overall body motion characterized by the orbital and local motion about the center of mass. This motion is related to the observations made by ground stations and provides orientation of the total motion in the antenna axes systems and orientation of the lines of sight relative to the ground.

Since only the basic characteristic motions are required and exacting simulation to some existing satellite is not necessary, certain simplifications are introduced. These include omission of perturbations and observational errors and the elimination of a true time requirement and a time based progression reference.

The emphasis here is on a description of the equations, relations, and transformations of the simulation model. The computer program based on this simulation model is described in a separate report.

Acknowledgement

The author is indebted to S. H. Bickel for his useful discussions and suggestings.

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1.0 INTRODUCTION

The overall motion of a satellite is characterized by its orbital motion and local motion about its center of mass. It is of interest to discern both these motions from ground viewing stations. Thus it is desired to relate the motion to observations at a complex of earth stations each of which totally positions the body in space relative to its line of sight.

For a given system and operation mode, correlation of the observations to the body motion can provide a method for developing signature analysis techniques for describing body configuration, orientation, and motion. Simulation of the body motion and measuring techniques provides a convenient approach for carrying out such a study.

This paper will be restricted to a discussion of the simulation of the body motion only. It will relate this motion as a final output to a receiving antenna axis system along the line of sight. What is discussed for one site carries over to any other site.

The equations, relations, and transformations required in the consideration of overall motion have been considered in earlier internal reports by the author. The latter reference includes a simulation of the resulting scattering matrix from stored scattering data and sets up radar outputs for various polarization modes. This radar simulation also includes the effect of Faraday rotation and noise.

In a computer program many variations are possible. These include differences in describing the station, orbit, and their combination for observation. The present paper presents a somewhat simplified version, as compared to earlier reports by the author, of the motion simulation.†

Since only the basic motions are required, no perturbations or observational errors are included. Again, since it is not the intention here to simulate motion of any existing satellite as close as possible, accounting for true time, taking time as a progression reference, and solving the transcendental Kepler equation (see section

† A description of the program, already completed, will be given in a subsequent report.

on orbital motion) also become unnecessary.

Thus, using a revolving spherical earth and earth surface fixed site locations (earlier internal reports of the author comment on ellipsoidal earth and nonzero altitude stations), the orientation of body fixed coordinates with respect to antenna coordinates are determined with time. These determinations are specified in terms of three orientation angles. In addition, the orientation of the station line of sight relative to the ground and the station to body range are provided. In the radar case, these angles are used to look up stored scattering data obtained either from measurement or calculation.

Both stabilized and unstabilized motion of the body are considered. It is in the latter, no torque, case that the simulation of local motion enters. The stabilized modes of major importance, spin stabilized, horizon stabilized, and earth centered stabilized, are accounted for by establishing the proper orientation.

2.0 THE SIMULATION MODEL

The basic technique of the simulation involves a sequence of six coordinate transformations. The coordinate systems dealt with are designated in order as follows.

1.	geometrical body axes,	$\{b'\}$, describes body geometry and symmetries†
2.	inertial axes,	$\{b\}$, describes conveniently body motion about center of mass
3.	angular momentum,	$\{\bar{\xi}\}$, used as a reference with respect to body local motion
4.	orbital,	$\{\xi\}$, used as a reference with respect to earth motion
5.	geocentric,	$\{g\}$, used as a reference with respect to earth motion
6.	topocentric,	$\{s\}$, moves with earth and locates the site(s)
7.	radar,	$\{r\}$, moves with radar and accounts for antenna orientation

---

† In the radar case, this provides a reference to which scattering data is related in terms of a radar measurement coordinate system $\{r'\}$. Details are given in an internal report by the author as well as in reference 1.

With perturbations on orbital and local motion due to, for example, earth oblateness, drag, solar system influences, solar radiation, and gravity-magnetic torques omitted, the $\{g\}$, $\{\xi\}$, and $\{\tilde{\xi}\}$ systems are inertial.[†] Also a Keplerian orbit in a true square law force field is used. In calculating the earth rotation rate ω_E, account must be taken of the earth revolution which effectively increases the rotation of the $\{s\}$ frame with respect to the $\{g\}$ frame, for existing satellite simulation.

There are six linear transformations relating each coordinate system with the one which follows it in the list above. These transformations are thus designated:

$$T'_{b,b}, T_{b,\tilde{\xi}}, T_{\tilde{\xi},\xi}, T_{\xi,g}, T_{g,s}, T_{s,r}$$

The relation of the $\{b'\}$ system to the $\{b\}$ system can be specified or calculated from body mass data from diagonalization of the inertia tensor. $T_{b,\tilde{\xi}}$, $T_{g,s}$ and $T_{s,r}$ are time varying where $T_{b,\tilde{\xi}}$ carries the local motion, $T_{g,s}$ carries earth rotation and $T_{s,r}$ reflects orbital motion which is specified by the radius vector R_{ξ} in the $\{\xi\}$ system. Local motion is specified by movement of the $\{b\}$ inertial axis system with respect to the $\{\tilde{\xi}\}$ system. The overall orientation of the body axes $\{b'\}$ to the radar system $\{r\}$ is given by the product of linear transformations from which three orientation angles are determined.

$$T_{b',r} = T_{s,r}\, T_{g,s}\, T_{\xi,g}\, T_{\tilde{\xi},\xi}\, T_{b,\tilde{\xi}}\, T_{b',b}$$

A brief discussion related to the different transformations is appropriate prior to presenting the overall simulation logic.

Each of the transformations can be given with respect to a coordinate system as a 3 x 3 matrix, $T_{\alpha,\beta}$ relating the α system to the β system. More precisely, $T_{\alpha,\beta}$ converts the coordinates of a vector in the $\{\alpha\}$ to its coordinates in the $\{\beta\}$ system. $\tilde{T}_{\alpha,\beta}$, on the other hand, relates the basis vectors of the $\{\beta\}$ system from the basis vectors of the $\{\alpha\}$ system.

[†] Additional detail is given in an internal report by the author.

In general[†]

$$T_{\alpha,\beta} = (\widetilde{T}'_{\alpha,\beta})^{-1}$$

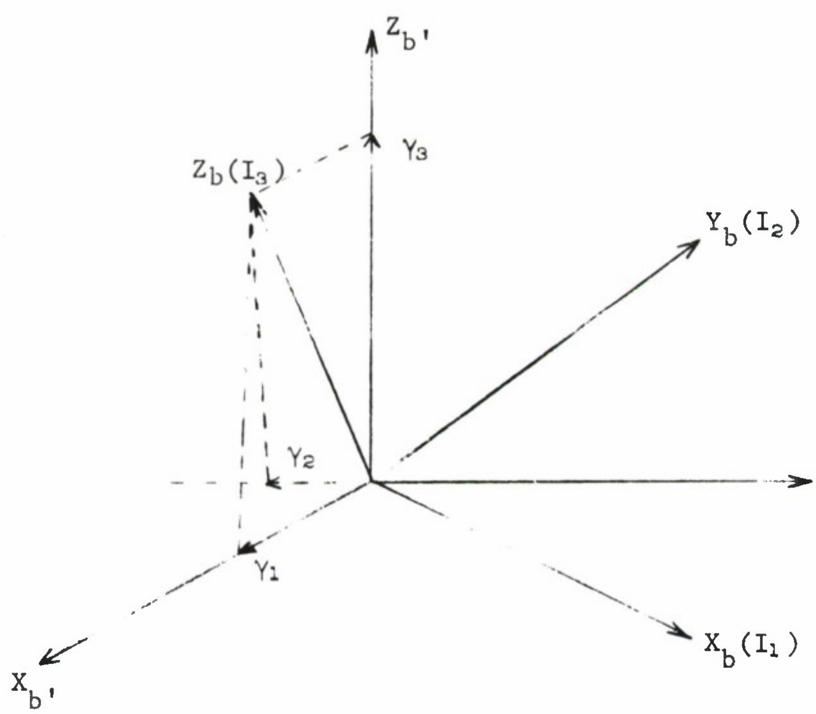
It is sometimes easier to consider the $T_{\alpha,\beta}$ in terms of the $\widetilde{T}_{\alpha,\beta}$ transformations as successive axes movements.

Certain of the $\widetilde{T}_{\alpha,\beta}$ are considered composed of factor transformations each of which are pure rotations (in some plane) $\widetilde{T}^{()}$, so that $(\widetilde{T}^{()})' = (\widetilde{T}^{()})^{-1}$.

Then

$$T_{\alpha,\beta} = \widetilde{T}_{\alpha,\beta} \qquad (\text{and so } T'_{\alpha,\beta} = T_{\alpha,\beta}^{-1})$$

$T_{b',b}$:



† Prime is transpose and ( )$^{-1}$ is inverse

$$T_{b',b} = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix} = (\alpha_1' \; \alpha_2' \; \alpha_3')^{\dagger}$$

α_1, α_2, α_3 are the direction cosines of X_b onto $\{b'\}$

β_1, β_2, β_3 are the direction cosines of Y_b onto $\{b'\}$

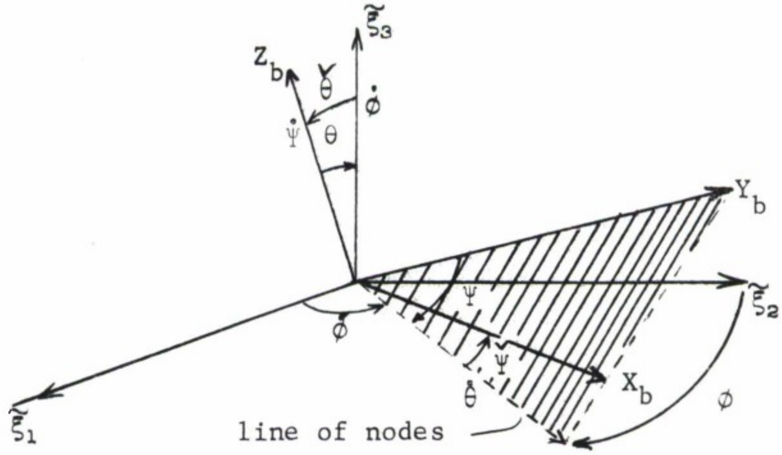
γ_1, γ_2, γ_3 are the direction cosines of Z_b onto $\{b'\}$

The I_1, I_2, I_3 signify the moments of inertia along the corresponding principal moments of inertia axes X_b, Y_b, and Z_b respectively. The $\{b'\}$ system has its coordinate axes along directions determined by body geometry and symmetries. In general the $\{b\}$ and $\{b'\}$ systems will not coincide. The specification of $T'_{b,b}$ is effected in a number of ways.

1. By inserting as input data,
2. By calculation from body mass and configuration data,
3. By specification as identity ($T_{b',b} = I$).

The calculation, if performed, is done so only in the torqueless, unstabilized mode. For all stabilized modes it is sufficient to consider no difference between the $\{b\}$ and $\{b'\}$ systems ($T_{b',b} = I$) with controlled orientations taken with respect to the $\{b'\}$ system.

$T_{b,\tilde{\xi}}$:



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† α_1' is a column vector with components α_1, β_1, γ_1 etc. Indeed α_1', α_2', α_3' are eigenvectors of the moment of inertia tensor.

$\tilde{\xi}_3$ is the direction of the angular momentum vector of local motion, fixed in inertial space. The $\tilde{\xi}_1$, $\tilde{\xi}_2$ vectors form a right handed $\{\tilde{\xi}\}$ system and are specified with respect to the $\{\tilde{\xi}\}$ as noted later.

The Euler angles θ, Ψ, ϕ measure the motion about the center of mass of the body inertia axes $\{b\}$ relative to the fixed in space system $\{\tilde{\xi}\}$. The line of nodes is the intersection of the (X_b, Y_b) plane and the $(\tilde{\xi}_1, \tilde{\xi}_2)$ plane. A description of the equation describing the Euler angles as functions of time is given below under the section local motion. We have

$$T_{b,\tilde{\xi}} = \tilde{T}_{b,\tilde{\xi}} = \tilde{T}^{(3)}_{b,\tilde{\xi}}(-\check{\phi})\, T^{(2)}_{b,\tilde{\xi}}(-\theta)\, T^{(1)}_{b,\tilde{\xi}}(-\check{\Psi}) \qquad \begin{aligned}\check{\Psi} &= -\Psi - \pi/2\\ \check{\phi} &= \phi + \pi/2\,.\end{aligned}$$

$$= \begin{pmatrix} \cos(-\check{\phi}) & \sin(-\check{\phi}) & 0 \\ -\sin(-\check{\phi}) & \cos(-\check{\phi}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(-\check{\theta}) & \sin(-\check{\theta}) \\ 0 & -\sin(-\check{\theta}) & \cos(-\check{\theta}) \end{pmatrix} \begin{pmatrix} \cos(-\check{\Psi}) & \sin(-\check{\Psi}) & 0 \\ -\sin(-\check{\Psi}) & \cos(-\check{\Psi}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\sin\phi & -\cos\phi & 0 \\ \cos\phi & -\sin\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -\sin\Psi & \cos\Psi & 0 \\ -\cos\Psi & -\sin\Psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^{(1)}_{b,\tilde{\xi}}(-\check{\Psi}) \quad \text{takes} \quad \begin{Bmatrix} X_b \\ Y_b \\ Z_b \end{Bmatrix} \quad \text{into} \quad \begin{Bmatrix} X'_b \\ Y'_b \\ Z_b \end{Bmatrix}$$

$$T^{(2)}_{b,\tilde{\xi}}(-\check{\theta}) \quad \text{takes} \quad \begin{Bmatrix} X'_b \\ Y'_b \\ Z_b \end{Bmatrix} \quad \text{into} \quad \begin{Bmatrix} X'_b \\ Y''_b \\ \tilde{\xi}_3 \end{Bmatrix}$$

$$T^{(3)}_{b,\tilde{\xi}}(-\check{\phi}) \quad \text{takes} \quad \begin{Bmatrix} X'_b \\ Y''_b \\ \tilde{\xi}_3 \end{Bmatrix} \quad \text{into} \quad \begin{Bmatrix} \tilde{\xi}_1 \\ \tilde{\xi}_2 \\ \tilde{\xi}_3 \end{Bmatrix}$$

---

† In general the angle between X_b and $\tilde{\xi}_1 \to \phi + \Psi$ as $\theta \to 0$.

For spin stabilized mode, $Z_{b'} = Z_b$ is fixed in the $\{\xi\}$ system and $\dot{\Psi} = \dot{\phi}$ (the spin rate about Z_b') is fixed.

Then $T_{b,\tilde{\xi}} = \begin{pmatrix} \cos \check{\Psi} & -\sin \check{\Psi} & 0 \\ \sin \check{\Psi} & \cos \check{\Psi} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

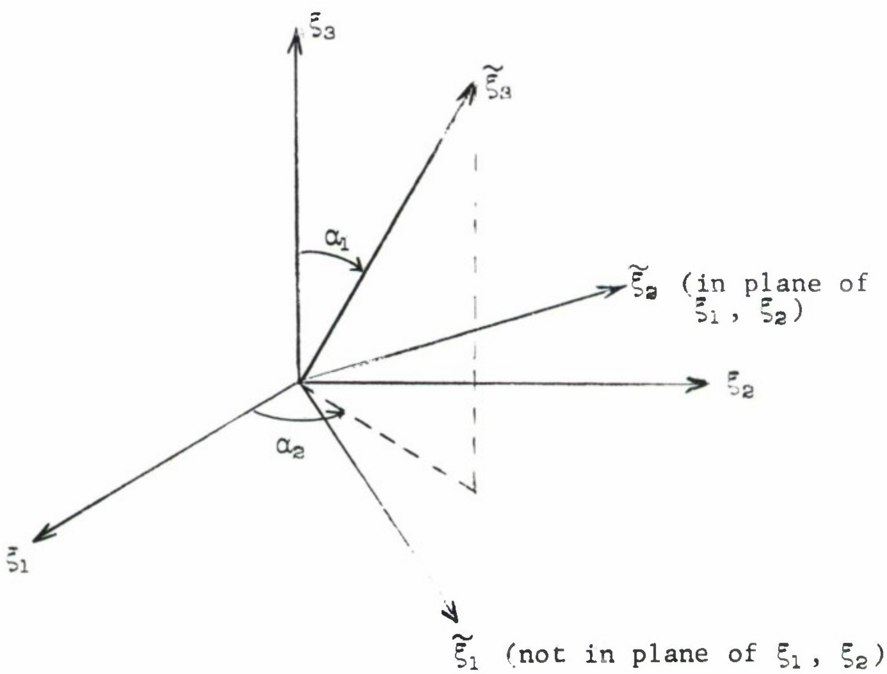
(i.e., $\check{\theta} = 0$, $\check{\phi} = 0$, $\check{\Psi} = \check{\Psi}_o + \dot{\Psi}\Delta t$)

For horizon stabilized mode, $Z_{b'} = Z_b$ is parallel to $\dot{R}_\xi$ and thus the $\{\tilde{\xi}\}$ system is bypassed and

$$T_{b,\tilde{\xi}} = I \qquad (\text{i.e., } \check{\theta} = \check{\phi} = \check{\Psi} = 0)$$

For earth centered stabilized mode, $Z_{b'} = Z_b$ is parallel to R_ξ and thus similarly, $T_{b,\tilde{\xi}} = I$ (i.e., $\check{\theta} = \check{\phi} = \check{\Psi} = 0$)

$T_{\tilde{\xi},\xi}$:



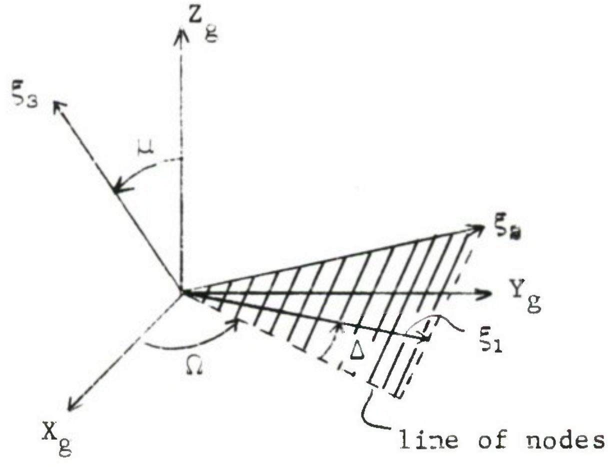
Since rotation of the $(\tilde{\xi}_1, \tilde{\xi}_2)$ plane about $\tilde{\xi}_3$ is arbitrary, we choose $\tilde{T}_{\tilde{\xi},\xi}$ to be given only by the two angles α_1 and α_2. (In fact, $\phi(0)$, which measures the initial rotation of the $\{b\}$ system about $\tilde{\xi}_3$, is specified and arbitrary in any case.) ξ_1 is in the direction of perigee from the $\{\xi\}$ origin (earth center), ξ_3 is normal to the orbit plane, and ξ_2 forms a right handed coordinate inertial system.

$$T_{\tilde{\xi},\xi} = \tilde{T}_{\tilde{\xi},\xi} = \tilde{T}_{\tilde{\xi},\xi}^{(2)}(-\alpha_2)\,\tilde{T}_{\tilde{\xi},\xi}^{(1)}(-\alpha_1) = \begin{pmatrix} \cos\alpha_2 & -\sin\alpha_2 & 0 \\ \sin\alpha_2 & \cos\alpha_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \cos\alpha_1 & 0 & -\sin\alpha_1 \\ 0 & 1 & 0 \\ \sin\alpha_1 & 0 & \cos\alpha_1 \end{pmatrix}$$

For unstabilized motion α_1 and α_2 are specified as input.
For spin stabilized motion α_1 and α_2 are specified as input,
For horizon stabilized motion $\alpha_1 = \pi/2$ is specified and $\alpha_2 = \phi_2$
(places Z_b, in parallel with $\dot{R}_\xi$)
For earth centered stabilized $\alpha_1 = \pi/2$ is specified and $\alpha_2 = \phi_1$
(places Z_b, in parallel with R_ξ)

The time variation of the radius vector of the orbiting body, R_ξ, and the determination of the angles ϕ_1 and ϕ_2 are given below in the section on orbital motion.

$T_{\xi,g}$:



The geocentric coordinate system $\{g\}$ is chosen in an arbitrary way since initial time is of no consequence for the purpose of this simulation. In other words, the X_g direction is taken as the direction in space made by the earth radius passing through the equator at 0^o longitude at an arbitrary initial time, t_o, which is taken as zero. The Z_g is the direction from earth center through the north pole at t_o. Y_g completes a right handed inertial system. With no perturbations μ, Δ, and Ω are time invariant where:

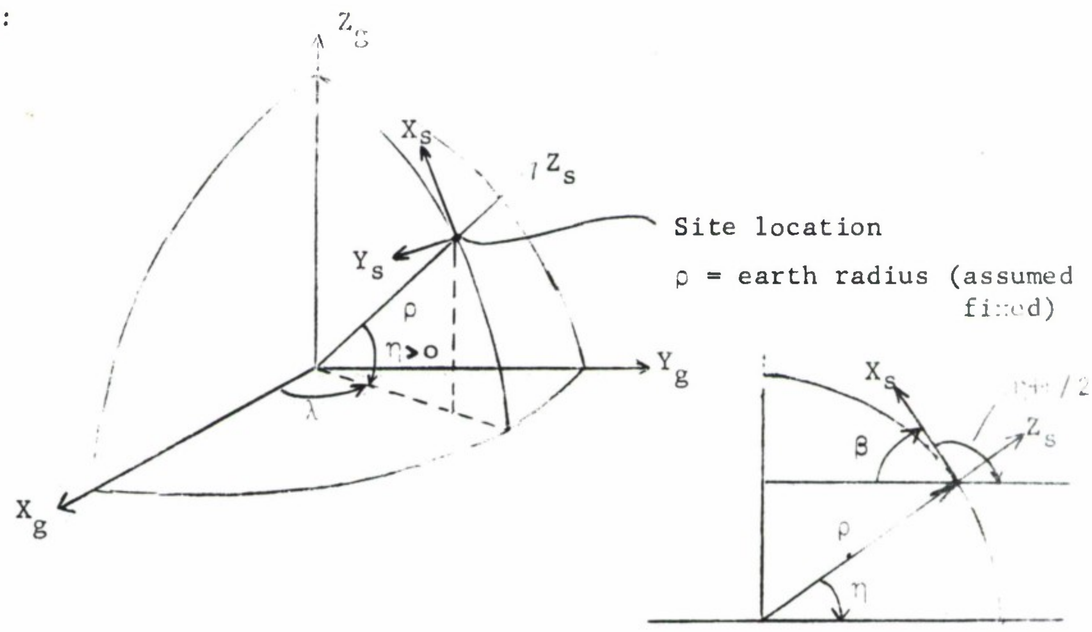
μ = inclination

Δ = argument of perigee †

Ω = right ascension of ascending node

$$T_{\xi,g} = \tilde{T}_{\xi,g} = \begin{pmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\mu & -\sin\mu \\ 0 & \sin\mu & \cos\mu \end{pmatrix} \begin{pmatrix} \cos\Delta & -\sin\Delta & 0 \\ \sin\Delta & \cos\Delta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$T_{g,s}$:



$(-\beta+\eta+\pi/2 = \pi)$

---

† Perigee or apogee are convenient choices of orbit epochs in an orbit period.

The Z_s direction is normal to earth surface pointing away from the earth at the site while X_s and Y_s determine the tangent plane at the site with X_s along a meridian to the north and Y_s pointing west where

$$\lambda = \text{longitude of site} \qquad (0 \le \lambda \le 2\pi)$$

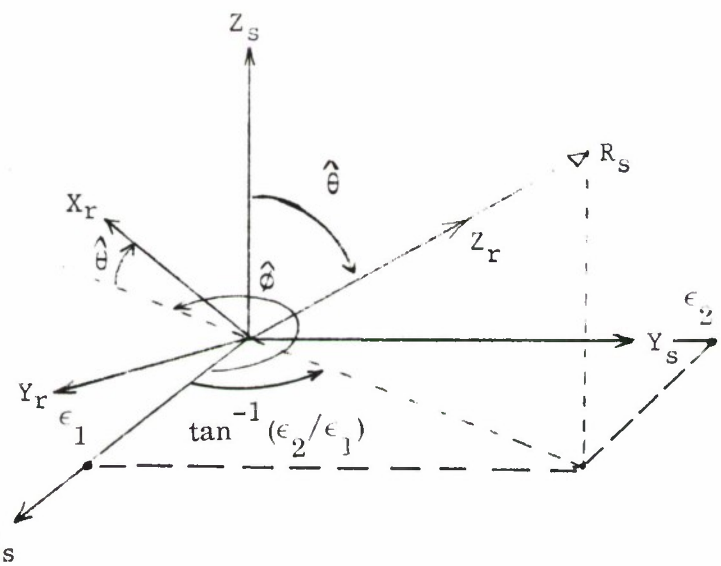
$$\eta = \text{measures latitude of site} \qquad (-\pi/2 \le \eta \le \pi/2)$$

$$T_{g,s} = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $\alpha = \omega_E \Delta t + \lambda + \pi$; $\quad \Delta t = t - t_o$
(with t_o arbitrary we usually take $t_o = 0$)†

$$\beta = \eta - \pi/2$$

$T_{s,r}$:



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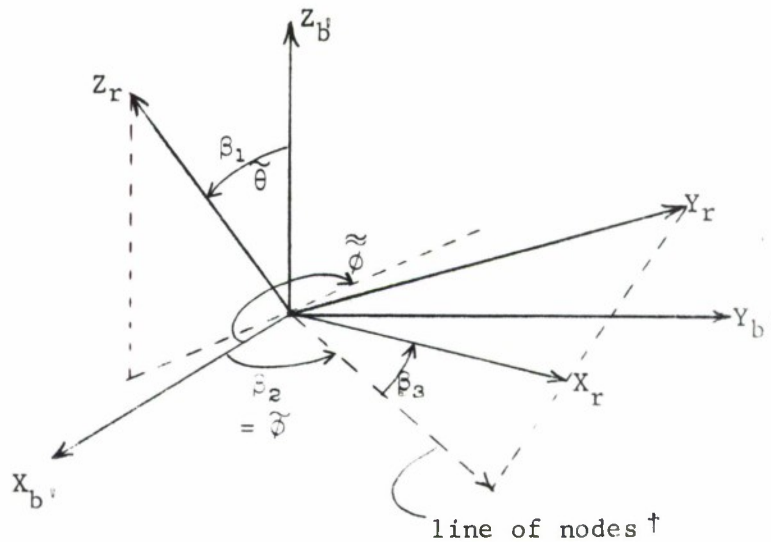
† With simulation of actual orbits, fixing $\{g\}$ in space at t_o consistent with astronomical data would mean, for example, taking t_o as vernal equinox time.

R_s is the radius (line of sight) vector to the orbiting body expre... in the $\{s\}$ system. R_s and the look angles $\hat{\theta}$ and $\hat{\phi}$ will be consi... in the section on orbital motion.

The Y_r of the radar system lies in the (X_s, Y_s) plane, that i... remains horizontal. It then is the axis about which elevation, $\hat{\theta}$, i... changed. $\hat{\phi}$ measures azimuth change.

$$T_{s,r} = \begin{pmatrix} \cos\hat{\theta} & 0 & \sin\hat{\theta} \\ 0 & 1 & 0 \\ -\sin\hat{\theta} & 0 & \cos\hat{\theta} \end{pmatrix} \begin{pmatrix} \cos\hat{\phi} & \sin\hat{\phi} & 0 \\ -\sin\hat{\phi} & \cos\hat{\phi} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$T_{b',r}$:



$$T_{b',r} = T_{s,r}\, T_{g,s}\, T_{\xi,g}\, T_{\tilde{\xi},\xi}\, T_{b,\tilde{\xi}}\, T_{b',b}$$

---

† In the radar case, the line of nodes coincides with the $X_{r'}$ of the radar measuring system $\{r'\}$ which is a rotation of $\{r\}$ about Z_r of $-\beta_3$. Y_r and $Y_{r'}$ are horizontal axes in their respective systems. $\tilde{\theta}$ and $\tilde{\phi}$ are drawn to be consistent with an adopted use in the $\{r'\}$ system for scattering measurements (see reference 1).

$$T_{b;r} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\beta_3 & \sin\beta_3 & 0 \\ -\sin\beta_3 & \cos\beta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta_1 & \sin\beta_1 \\ 0 & -\sin\beta_1 & \cos\beta_1 \end{pmatrix} \begin{pmatrix} \cos\beta_2 & \sin\beta_2 & 0 \\ -\sin\beta_2 & \cos\beta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This gives

$a_{11} = \cos\beta_3 \cos\beta_2 - \sin\beta_3 \cos\beta_1 \sin\beta_2$

$a_{21} = -\sin\beta_3 \cos\beta_2 - \cos\beta_3 \cos\beta_1 \sin\beta_2$

$a_{31} = \sin\beta_1 \sin\beta_2$

$a_{12} = \cos\beta_3 \sin\beta_2 + \sin\beta_3 \cos\beta_1 \cos\beta_2$

$a_{22} = -\sin\beta_3 \sin\beta_2 + \cos\beta_3 \cos\beta_1 \cos\beta_2$

$a_{32} = -\sin\beta_1 \cos\beta_2$

$a_{13} = \sin\beta_3 \sin\beta_1$

$a_{23} = \cos\beta_3 \sin\beta_1$

$a_{33} = \cos\beta_1$

Then $\beta_1 = \tan^{-1}\left[\dfrac{(1-a_{33}^2)^{\frac{1}{2}}}{a_{33}}\right] = \cos^{-1} a_{33}$

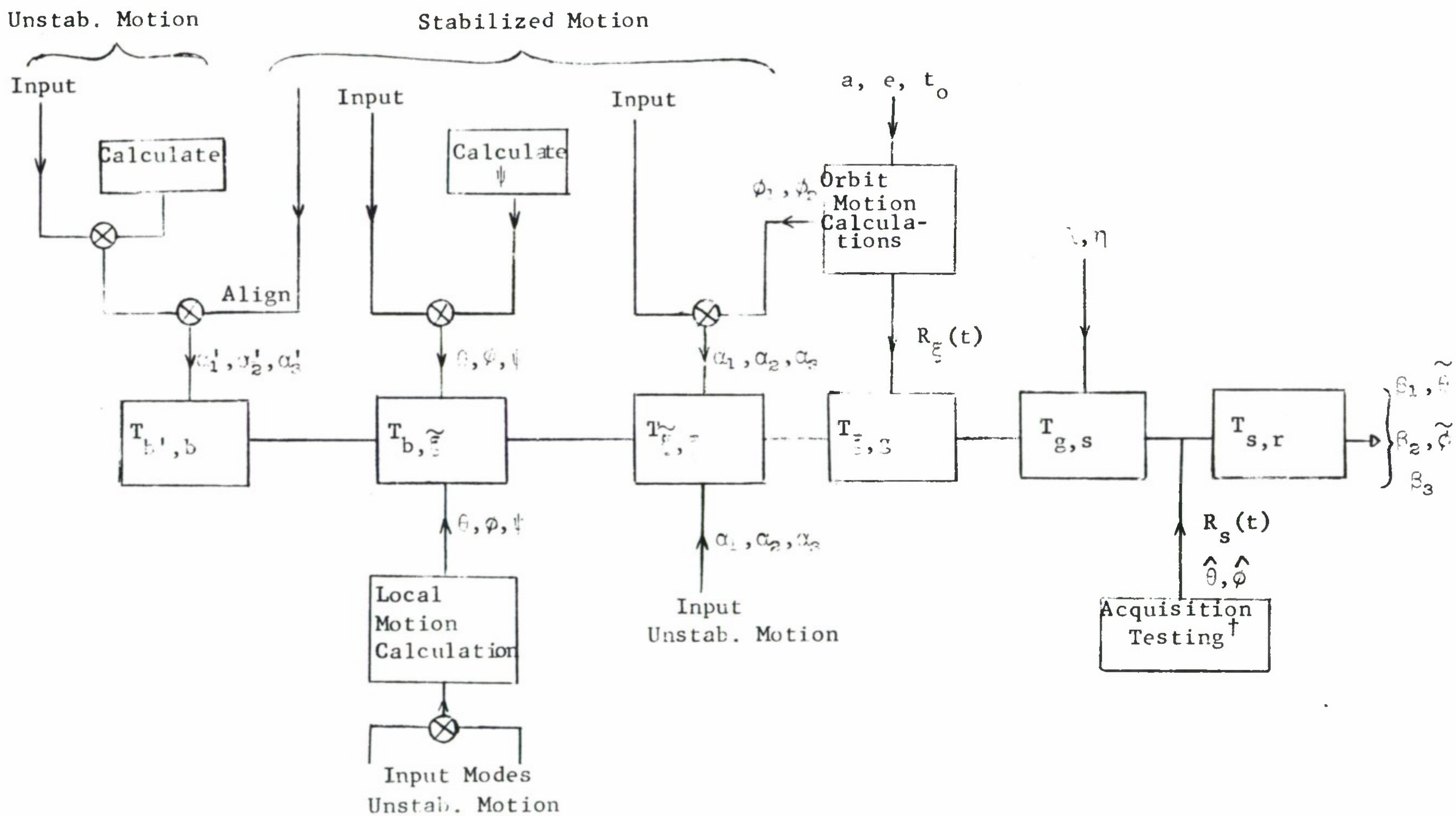
$$\beta_2 = \tan^{-1}\left(\frac{a_{31}}{-a_{32}}\right)$$

$$\beta_3 = \tan^{-1}\left(\frac{a_{13}}{a_{23}}\right)$$

$\tilde{\theta} = \beta_1, -\tilde{\tilde{\phi}} = (3\pi/2 - \beta_2),\ \tilde{\phi} = \beta_2$

$\tilde{\theta}$, $\tilde{\phi}$, and β_3 are the final antenna body orientation angles.

The overall simulation logic can now be conveniently displayed schematically by the following diagram. Of course either stabilized motion or local motion is appropriate. For convenience both are shown. The



Overall Simulation Logic Diagram

† The acquisition testing checks to see that $\hat{\theta}$ is such as to allow the station to view the target. When contact is lost the next position of visibility is determined.

symbol ⊗ designates alternate paths. The logic flow chart for the actual program appears in a subsequent report.

3.0 ORBITAL MOTION

With an inverse square law radial force field from earth center, the equation for motion in the $\{\xi\}$ system becomes

$$\ddot{R}_\xi + d \frac{R_\xi}{|R_\xi|} = 0$$

with d = constant, accounting for gravity and the mass of the two body system. In the metric system, for example

$$d = 3.9858 \times 10^5 \ \mathrm{km}^3/\mathrm{sec}^2.$$

This results in planar motion, that is $R \times \dot{R}$ = constant so that

$R = a\,R_o + b\,\dot{R}_o$; with a and b scalar functions of time and

R_o and $\dot{R}_o$ evaluations at some time t_o'.

Choosing t_o' to be the time of perigee and the $\{\xi\}$ coordinate system then places R_o along ξ_1, $\dot{R}_o$ along ξ_2 and gives

$$R_\xi(t) = a(\cos E - e)\,\xi_1 + a(1-e^2)^{\frac{1}{2}} \sin E\,\xi_2$$

and $$\dot{R}_\xi(t) = -a\,\dot{E} \sin E\,\xi_1 + a(1-e^2)^{\frac{1}{2}}\,\dot{E} \cos E\,\xi_2$$

where a is the semimajor axis of the elliptical orbit and e is the ellipticity of the orbit. E(t), the eccentric anomoly is related to t in Kepler's equation

$$E - e \sin E = (d/a^3)^{\frac{1}{2}} \Delta t \ ; \ \Delta t = t - t_o'$$

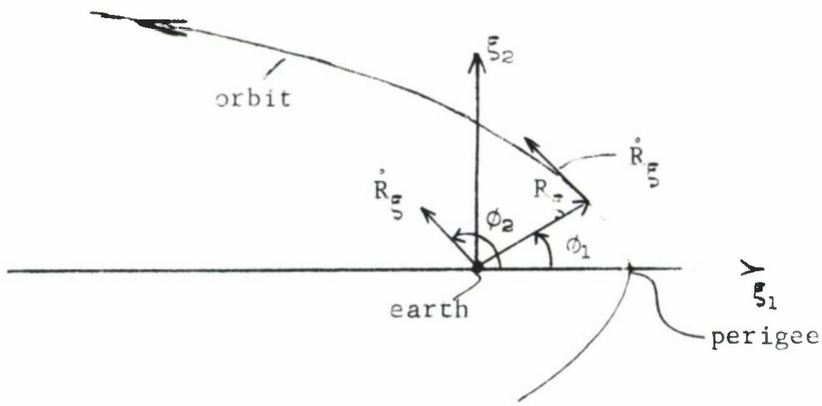
$$(\dot{E} = (d/a^3)^{\frac{1}{2}} / (1 - e \cos E))$$

where t_o' is time of perigee crossing. If initial time t_o is not t_o' the Kepler equation is slightly more complex but straightforward. We take $t_o = t_o'$. Also equal increments in Δt produce unequal increments in E and vice versa. Solving for E from t is difficult whereas solving for t from E is direct. The programmed version discussed in a subsequent working paper uses equal increments in E as reference. For simulations of actual orbits where motion is referenced to real times, solution of the Kepler equation for E can be accomplished using an iterative scheme such as Newton-Raphson.+

From the expressions for R_ξ and $\mathring{R}_\xi$ we have

$$\phi_1 = \tan^{-1}\left[\frac{(1-e^2)^{\frac{1}{2}}\sin E}{\cos E - e}\right]$$

$$\phi_2 = \tan^{-1}\left[\frac{-(1-e^2)^{\frac{1}{2}}}{\tan E}\right]$$



+ Let $f(E) = (E - e\sin E) - A$; $A(t) = (d/a^3)^{\frac{1}{2}}(t - t_o)$

$\mathring{f}(E) = df/dE = 1 - e\cos E$

With E_n the n^{th} estimate of true E, let h_n be such that

$$f(E_n) + h_n\,\mathring{f}(E_n) = 0, \text{ i.e., } h_n = -\frac{f(E_n)}{\mathring{f}(E_n)}$$

We take $E_{n+1} = h_n + E_n$ and repeat the process for h_{n+1} until $|f(E_n)| < \epsilon$, a preselected small number.

Now $R_s(t) = T_{g,s}\, T_{\xi,g}\, R_{\xi} - \rho$

$$= \epsilon_1 X_s + \epsilon_2 Y_s + \epsilon_3 Z_s$$

where as noted before, ρ is earth radius taken away from earth center.

$$|R_s| = (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)^{\frac{1}{2}}$$

then $$\hat{\theta} = \tan^{-1}\left[\frac{(\epsilon_1^2 + \epsilon_2^2)^{\frac{1}{2}}}{\epsilon_3}\right] \qquad \epsilon_1 = R_s \cdot X_s \text{ etc.}$$

$$\hat{\phi} = \pi + \tan^{-1}\left(\frac{\epsilon_2}{\epsilon_1}\right)$$

The geometry of $\hat{\theta}$ and $\hat{\phi}$ was shown in the discussion of $T_{s,r}$ in the previous section.†

Of various acquisition tests a useful and simple scheme compares $\hat{\theta}$ against some $\hat{\theta}_1$ such that

if (i) $\hat{\theta} < \hat{\theta}_1$, target visible

(ii) $\hat{\theta} \geq \hat{\theta}_1$, target not visible

If (ii) holds, proceed along orbit until (i) holds.
If (i) holds, continue with main calculations.

4.0 LOCAL MOTION

The movement of any vector fixed in the body and so moving with the $\{b\}$ system with respect to the $\{\tilde{\xi}\}$ system can be described by the three Euler angles $\Theta(t)$, $\Psi(t)$, and $\phi(t)$ defined in the discussion of $T_{b,\tilde{\xi}}$. Indeed, these angles satisfy the following Euler differential equations, for zero applied torque.

† We call the plunged condition of the radar with $\hat{\phi} + \pi$ rather than $\hat{\phi}$ (also possible). Then for X_r above the (X_s, Y_s) plane in the normal condition, X_r and R_s have projections in the plane in opposite directions whereas in the plunged condition X_r is below the (X_s, Y_s) plane and the two projections lie in the same direction.

$$\dot{\theta} = \frac{(I_1 - I_2)}{I_1 \; I_2} = M \sin\theta \cos\Psi \sin\Psi = \text{nutation rate (for small } \Delta\theta \text{ variations)}$$

$$\dot{\phi} = \frac{M}{I_1} \cos^2 \Psi + \frac{M}{I_2} \sin^2 \Psi = \text{precession rate}$$

$$\dot{\Psi} = \left(\frac{M}{I_3} - \frac{M}{I_1} \cos^2 \Psi - \frac{M}{I_2} \sin^2 \Psi \right) \cos\theta = \text{spin rate}$$

where for generality† we take $I_1 \geq I_2 \geq I_3$ as the three moments of inertia with respect to the principal axes of inertia X_b, Y_b, Z_b, and M is the magnitude of the angular momentum, $\vec{M}$ along $\tilde{\xi}_3$.

Denoting the corresponding angular velocities as $\vec{\Omega}_1(t)$, $\vec{\Omega}_2(t)$ and $\vec{\Omega}_3(t)$ with directions along the X_b, Y_b, Z_b axes we have

$$\theta(0) = \cos^{-1}\left[\frac{I_3 \; \Omega_3(0)}{M}\right], \qquad \Psi(0) = \tan^{-1}\left[\frac{I_2 \; \Omega_2(0)}{I_1 \; \Omega_1(0)}\right]$$

and $\vec{M} = \sum_{i=1}^{3} I_i \; \Omega_i$, $\qquad M^2 = \sum_{i=1}^{3} I_i^2 \; \Omega_i^2$,

$\phi(0)$ is arbitrary and is specified. Note initial values of θ and Ψ reflect initial values of $\dot{\theta}$, $\dot{\phi}$, and $\dot{\Psi}$.

The solutions to the Euler equations (see reference 2) are obtained in series which allow for approximations to any desired accuracy. These solutions are: ††

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† In all cases each moment of inertia along an axis is less than or equal to the sum of the moments of inertia along the other two orthogonal axes. For definetness the inequalities are used.

†† In order that the expressions for $\theta(t)$, $\Psi(t)$, and $\phi(t)$ be consistent with initial conditions, the t used in these expressions should be shifted with respect to true (orbit) time.

$$\theta(t) = \cos^{-1}\left\{\frac{(\sinh\gamma - q^2\sinh 3\gamma + \cdots)(1+2q\cos 2\mu t + 2q^4\cos 4\mu t + \cdots)}{(\cosh\gamma + q^2\cosh 3\gamma + \cdots)(1-2q\cos 2\mu t + 2q^4\cos 4\mu t - \cdots)}\right\}$$

$$\text{from } \cos\theta = \frac{\vartheta_{11}(ia/2K)\ \vartheta_{00}(\lambda t/2K)}{i\vartheta_{10}(ia/2K)\ \vartheta_{01}(\lambda t/2K)}$$

(ϑ_{xx} () are the four types of theta functions and K is the complete elliptic integral of the first kind.)

$$\Psi(t) = \tan^{-1}\left\{\frac{(1+2q\cosh 2\gamma + 2q^4\cosh 4\gamma + \cdots)(\sin\mu t - q^2\sin 3\mu t + \cdots)}{(1-2q^2\cosh 2\gamma + 2q^4\cosh 4\gamma - \cdots)(\cos\mu t + q^2\cos 3\mu t + \cdots)}\right\}$$

$$\text{from } \tan\Psi = \frac{\vartheta_{00}(ia/2K)\ \vartheta_{11}(\lambda t/2K)}{\vartheta_{01}(ia/2K)\ \vartheta_{10}(\lambda t/2K)}$$

$$e^{2i\phi(t)} = F(t)\exp\left\{2i\left[\frac{M}{I_1} + 4\mu\left(\frac{q\sinh 2\gamma - 2q^4\sinh 4\gamma + \cdots}{1-2q\cosh 2\gamma + 2q^4\cosh 4\gamma + \cdots}\right)\right]t\right\}$$

$$= F(t)\exp\left\{2i\left[\frac{M}{I_1} + \frac{\lambda}{2iK}\frac{\vartheta'_{01}(ia/2K)}{\vartheta_{01}(ia/2K)}\right]t\right\}$$

where

$q = k^2/16 + k^4/32 + \cdots$; $q^2 = e^{-\pi K'/K}$; modulus of K is k.

$$(k')^2 = \left(\frac{I_1 - I_2}{I_2 - I_3}\right)\frac{(M^2 - I_3 C)}{(I_1 C - M^2)}; \qquad C = \sum_{i=1}^{3} I_i\,\Omega_i^2$$

= modulus of $K' = 1 - k^2$

$$\mu = \frac{\pi\lambda}{2K}, \quad \lambda^2 = \frac{(I_2 - I_3)(CI_1 - M^2)}{I_1 I_2 I_3}$$

$$\gamma = \pi a/K, \quad \left(\frac{2K}{\pi}\right)^{\frac{1}{2}} = 1 + 2q + 2q^4 + 2q^9 + \cdots$$

$$\left\{\frac{I_2(I_1C - M^2)}{I_1(I_2C - M^2)}\right\} = \frac{\vartheta_{00}(ia/2K)}{\vartheta_{01}(ia/2K)}$$

$$F(t) = B\,\frac{\vartheta_{01}\left(\frac{\lambda t - ia}{2K}\right)}{\vartheta_{01}\left(\frac{\lambda t + ia}{2K}\right)}$$

B is a constant specified by $\phi(0)$.

Together with $\dot{\phi}(0)$, q, μ, and γ represent a possibly more convenient input set than I_i, Ω_i, i=1,3. Indeed for q << 1, a meaningful association gives q as a measure of nutation amplitude, $\Delta\theta$, μ as a measure of spin rate, $\dot{\Psi}$, and γ as a measure of precession angle, θ. Writing

$$e^{2i\phi(t)} = e^{2i(\phi_1 + \phi_2)}$$

with $e^{2i\phi_1} = F(t)$, $\quad \phi_2 = \left(\frac{M}{I_1} + \frac{\lambda}{2iK} \frac{\vartheta'_{01}(ia/2K)}{\vartheta_{01}(ia/2K)} \right) t$

$$\left(\vartheta'() = \frac{d\vartheta()}{d()} \right)$$

then $\dot{\phi}_2(t)$ = constant, measures average rate of ϕ variation (precession rate) and $\phi_1(t)$ accounts for (periodic) variations about the average $\dot{\phi}$ $(=\dot{\phi}_2)$.

The frequency of the θ and Ψ periodicities $= \frac{\mu}{2\pi}$.

The frequency of the ϕ_1 component of $\phi = \frac{\mu}{\pi}$.

The frequency of the ϕ_2 component of $\phi = \pi\dot{\phi}_2$.

Various interesting special cases occur. For example constant precession ($\phi_1 = 0$) is obtained about $\tilde{\xi}_3$ with spin about Z_b when $I_1 = I_2$. (See reference 1.) For unstabilized bodies the particular motions of interest are normally tumbling, precession and nutation. In the long (time) term, the dominant motion is usually tumbling. All of these motions represent simplifications of the general motion.

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13. ABSTRACT

The application of simulation techniques provides a convenient and useful means for studying and developing signature analysis schemes to describe orbiting body configuration, orientation and motion.

This paper discusses a simulation of the overall body motion characterized by the orbital and local motion about the center of mass. This motion is related to the observations made by ground stations and provides orientation of the total motion in the antenna axes systems and orientation of the lines of sight relative to the ground.

Since only the basic characteristic motions are required and exacting simulation to some existing satellite is not necessary, certain simplifications are introduced. These include omission of perturbations and observational errors and the elimination of a true time requirement and a time based progression reference.

The emphasis here is on a description of the equations, relations, and transformations of the simulation model. The computer program based on this simulation model is described in a separate report.

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